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One establishes oneself within science from the start. One does not reconstitute it from scratch. One does not found it.

Alain Badiou, Le Concept de modèle[1]

[T]here are no crises within science, nor can there be, for science is the pure affirmation of difference.

Alain Badiou, "Marque et manque"[2]

ray brassier

BADIOU'S MATERIALIST EPISTEMOLOGY OF MATHEMATICS

i introduction

Throughout Badiou's work, mathematics enjoys a privileged status as paradigm of science and of "scientificity" in general. This has been a constant, from his first significant philosophical intervention, the 1966 article "The (Re)Commencement of Dialectical Materialism,"[3] notable for the way in which it already prefigures his subsequent (career-spanning) preoccupation with the relation between set-theory and category-theory, to his most recent work, wherein Badiou finally establishes a philosophical connection between these two branches of mathematics by arguing that the doctrine of being, laid out via set-theory in *Being and Event*[4] (1988), needs to be supplemented by a doctrine of appearance that mobilizes category-theory, as Badiou does in his forthcoming *Logics of Worlds*.[5] Two quotes, separated by over thirty years, are indicative of Badiou's unwavering commitment to the paradigmatically scientific status of mathematics. The first is from the aforementioned 1966 article: "[U]ltimately, in physics, fundamental biology, etc., mathematics is not subordinated and expressive, but primary and *productive*."[6] We shall try to explain what this primacy of mathematical "productivity" entails for Badiou by examining his early attempt to develop a "materialist epistemology" of mathematics in his first book, *The Concept of Model* (1969). But first we shall consider a second quote, from an interview with Peter Hallward in 1998: "In the final analysis, physics, which is to say the theory of matter, is mathematical. It is mathematical because, as the theory of the most objectified strata of the presented as such, it necessarily catches hold of being-as-being through its mathematicity."[7] This latter claim encapsulates an argument about the relation between mathematical ontology and the natural sciences implicit (though never explicitly articulated) in Badiou's most ambitious work to date, *Being and Event*. Summarizing very briefly, we can say the following: for Badiou axiomatic set-theory is the science of being as sheer multiplicity, the science of the presentation of presentation

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(rather than of *what* is presented); in other words, the science that guarantees access to presented reality.[8] Thus not only does the Zermelo–Fraenkel axiomatization of set-theory provide the fundamental infrastructure to which all of mathematics can be reduced, it is also the guarantor for the mathematical sciences' access to reality. Consequently, the implication would seem to be that, for Badiou, the "scientificity" of a given science is directly proportional to its mathematization: science is "scientific" precisely to the extent that it is mathematical. By the same token, the less a science depends upon mathematical formalization, the less scientific it is. Hence Badiou's deferential nods towards physics and his notorious disdain for biology, "that wild empiricism disguised as science."[9] But surely the claim that mathematics provides the sciences with their ultimate horizon of scientificity is damning evidence of Badiou's stubborn adherence to an unreconstructed (not to say anachronistic) "Platonist" and "foundationalist" conception of science; one which privileges an "ideal" and "a priori" mathematical realm of scientificity over science's "empirical" and "material" dimensions, and attempts to ground the latter's access to reality in the former?[10]

Yet, as we shall see, it is precisely such foundationalism, as well as all such distinctions between a priori and a posteriori, ideal and real, formal and material, that Badiou explicitly sets out to undermine through the materialist epistemology of mathematics laid out in *The Concept of Model*. Badiou's materialist critique of the ideological substructure which tacitly governs the aforementioned continuum of distinctions (formal/material, ideal/real, a priori/a posteriori) common to empiricism and idealism (indeed, for Badiou, idealism is merely a *variant* of empiricism) furnishes the key required in order to make sense of his subsequent – and apparently startlingly eccentric – claims on behalf of the privileged status of mathematics in *Being and Event*. If it is difficult to extract from Badiou's work a "philosophy of mathematics" conforming to the norms of the academic sub-discipline of the same name, or to render some of his philosophical claims about science intelligible in terms of the debates that define the field known as "philosophy of science" (e.g., realism vs. instrumentalism, analysis and reductionism, the nature of induction, the status of scientific law, inference to the best explanation, etc.), this is not only because mathematics functions as a *synecdoche* for science in Badiou but also because his Platonist materialism challenges those empiricist doxas (principally the distinction between "formal" and "material" sciences) that, precisely because of their uninterrogated but constitutive role in the debates that characterize "philosophy of science," have filtered down into the branch of the latter known as "philosophy of mathematics."[11] In this regard, we shall find it instructive to contrast Badiou's materialist critique of what he regards as the ideological distinction between "real" and "ideal," common to empiricism and idealism, with Quine's celebrated intra-empiricist subversion of the analytic/synthetic distinction.

Early in *The Concept of Model*, Badiou, alluding to Althusser, reminds us that "To talk of science [la *science*] in the singular is an ideological symptom."[12] There are only sciences, in the plural. Yet the synecdochal status of mathematics vis-à-vis the sciences seems to engender a paradox whereby everything Badiou has to say about "the sciences" is encoded in his statements about one of them, mathematics. How can Badiou reconcile his materialism, which insists on the irreducible plurality of sciences, with his Platonism, which ascribes a paradigmatic status to mathematics? The answer lies in understanding how, for Badiou, Platonism is precisely what recuses the empiricist distinction between thought and object.[13] Mathematics is neither merely a formalist game, the arbitrary manipulation of intrinsically meaningless symbols, nor a quasi-supernatural mystery presided over by a select priesthood who enjoy a privileged vantage onto a transcendent realm of eternal objects. What singularizes mathematics as paradigm for science is rather the exemplary nature of its autonomous *productivity*. As we shall see, for Badiou, mathematical productivity (and a fortiori, scientific productivity) consists in cutting or differentiating the notational material upon which it operates. Science is the production of stratified differences. We shall dissolve the apparent contradiction between Badiou's ascription of a synecdochal status to mathematics vis-à-vis the sciences, and his insistence on the latter's essential plurality, by showing how for Badiou mathematics is not an a priori formal science grounding the empirical sciences' access to reality but rather the paradigmatic instance of a productive experimental praxis. This is the materialist dimension of Badiou's Platonism.

ii the formal and the empirical

In *The Concept of Model*, Badiou is operating under the aegis of two fundamental distinctions:

1. The Althusserian distinction between "historical materialism," understood as the Marxist *science* of history, and "dialectical materialism," understood as the latter's *philosophical* counterpart.[14]
2. Badiou's own distinction between ideological *notions*, philosophical *categories*, and scientific *concepts*.

Philosophy, constituted by its reactive and/or parasitic relation to scientific innovation on the one hand, and its subservience to dominant ideological interests on the other, is defined as the practice of an "impossible relation" between science and ideology.[15] For the most part, philosophy consists in the ideological envelopment of science: philosophical categories denote "inexistent" objects wherein concepts and notions are variously combined.[16] Informed by the Marxist science of history, the task of a materialist philosophy (as opposed to a "philosophy of matter," which merely synthesizes an inexistent category "matter" through the notional envelopment of physico-biological concepts) is to expose and critique the reactionary ideologies encoded in various "philosophizations" of science and to supplant them by materialist categories capable of being deployed in the service of revolutionary ideology (philosophy, according to a famous Althusserian slogan, being "the class struggle in theory").[17] Badiou's aim in *The Concept of Model* is to isolate the scientific – i.e., logico-mathematical – concept of model from its notional envelopment by the categories of bourgeois epistemology – central to which is the distinction between the "formal" and the "empirical" – and to construct a category of model consonant with a materialist history of the sciences.

We shall begin by recapitulating Badiou's critique of bourgeois epistemology. Ideological formations are structured as continuous combinations of variation on a difference whose principle is presupposed but never given in the series which it governs.[18] It is a characteristic of such formations that their notional variants are incapable of examining or legitimating their own underlying principle. The unthematized variational principle governing bourgeois epistemology is the notional difference between theoretical form and empirical reality: science is a formal representation of its object, whether the representation be characterized in terms of the effective "presence"[19] of the object, as is the case with empiricism, or in terms of the anteriority of a formal apparatus, i.e., of the mathematical code whereby the object is represented, as is the case with formalism (Badiou seems to have structuralism specifically in mind here). But in either case what must be borne in mind, Badiou insists, is that "[E]mpiricism and formalism have no other function here besides that of being the terms of the couple they form. What constitutes bourgeois epistemology is neither empiricism, nor formalism, but rather the set of notions through which one designates first their difference, then their correlation."[20] Thus the materialist critique of bourgeois epistemology must first identify the hidden theme, whose characteristic structure is that of a differential correlation between two opposed terms, governing the ideological continuum of notional variants. Badiou identifies a canonical variant of this theme in the opposition between Carnap and Quine regarding the status of the distinction between formal and empirical sciences. In "The Logical Foundations of the Unity of Science,"[21] Carnap begins by positing the difference between formal and empirical sciences before proceeding to seek rules of reduction governing the conversion of the terms of one empirical science into another. Carnap argues that biological terms are convertible into physical terms: Physics provides a sufficient basis for the reduction of biology.[22] Thus the language of science can be unified in so far as a "physicalist" language provides a universal basis for reduction for all the empirical sciences. Finally, Carnap's project – and more generally, the logical empiricist approach to the issue of the unity of science – culminates in the question of the relation between the fundamentally "physicalist" language of empirical science and the "artificial" languages of the formal sciences; in other words, the relation between the synthetic statements of the former and the analytical statements of the latter. But this is, of course, precisely the distinction that Quine calls into question in his celebrated 1951 article "Two Dogmas of Empiricism."[23] According to Quine, the notion of "analyticity" cannot withstand critical scrutiny: it relies on a notion of "synonymy," i.e., "sameness of meaning," which in turn presupposes a theoretically transparent account of the intensional dimension of "meaning." Quine challenges the intelligibility of the former and the possibility of the latter. While the notion of linguistic "extension" or reference can be rendered logically transparent, "intension" understood as a noematic entity tethered to the linguistic sign by noetic intention is simply "what [Aristotelian] essence becomes when it is divorced from the object of reference and wedded to the word."[24] Such a dubious metaphysical doctrine ("the 'idea' idea") cannot provide a reliable warrant for the notion of analyticity. Moreover, Quine goes on, the dogma of the analytic/synthetic distinction is indissociable from another empiricist dogma, the dogma of reductionism, manifested in Carnap's belief in the possibility of decomposing the truth of scientific statements into a formal or linguistic component on the one hand, and a factual or empirical one on the other. *Pace* Carnap, Quine insists on the intrinsically holistic character of the conceptual scheme called "science" and maintains that it is impossible to separate in it the contribution of language (i.e., conceptual convention) from the contribution of experience (empirical data):

> The totality of our so-called knowledge or beliefs, from the most casual matter of geography and history to the profoundest laws of atomic physics and even of pure mathematics and logic, is a man-made fabric which impinges upon experience only along the edges [. . .] But the total field is so underdetermined by its boundary conditions, experience, that there is much latitude of choice as to what statements to re-evaluate in the light of any single contrary experience. No particular experiences are linked with any particular statements in the interior of the field, except indirectly through considerations of the equilibrium affecting the field as a whole.[25]

Thus although acknowledging the underdetermination of scientific theory by empirical evidence, Quine refuses to abjure what Donald Davidson subsequently criticized as the "third" and ultimate dogma of empiricism: the dualism of conceptual scheme and empirical content.[26] In demolishing the analytic/synthetic distinction on empiricist grounds, Quine rejects Carnap's positivist "double standard" in the treatment of scientific hypotheses on one hand, and ontological questions on the other.[27] No hard-and-fast dividing line can be drawn to demarcate scientific hypothesizing from ontological speculation. Coupled with the Quinean doctrines of the indeterminacy of translation, which claims that reference is inscrutable unless relativized to a specific semantic coordinate system, and ontological relativity, which insists that "to be is to be the value of a variable," this scheme/content dualism leads Quine to embrace an epistemological relativism according to which the difference between Homeric gods and protons is merely one of degree rather than kind: "Both sorts of entities enter our conception only as cultural posits."[28] Whatever superiority the myth of physical objects enjoys over that of the Homeric gods comes down to a question of usefulness:

> As an empiricist, I continue to think of the conceptual scheme of science as a tool, ultimately, for predicting future experience in the light of past experience [. . .] The myth of physical objects is epistemologically superior to most in that it has proved more efficacious than other myths as a device for working a manageable structure into the flux of experience.[29]

Thus pragmatism is revealed as the truth of Quinean empiricism. Quine proposes to supplant Carnap's logical empiricism with a pragmatism which plainly exposes the point at which empiricism cannot but concede its own constitutive subordination to ideological imperatives whose status it is incapable of problematizing. If Quine sees no need for philosophy to investigate the mechanism of conceptual correlation whereby "cultural positing" is supposedly adjusted to "empirical usefulness," this is because the putative transparency of that mechanism reveals how empiricism remains conditioned by a set of ideological norms whose structure it cannot perceive. Thus the opposition between Carnap and Quine remains internal to an empiricist problematic structured around a difference between fact and form which neither can afford to question. In this regard, Quine's audacious subversion of the analytic/synthetic distinction obscures and perpetuates the more fundamental difference between formal and empirical which underlies it. Quine merely negates a distinction which Carnap seeks to reduce: "Whereas that reduction is essential to Carnap's discourse, all that matters in Quine's is the justification of the claim that it is not necessary to reduce what can be conveniently denied."[30] Quine's negation is convenient because it leaves empiricism's own ultimately empirical yet empirically imperceptible condition of possibility untouched. For Quine, as Badiou points out, it comes to the same thing whether one says that the empirical is a dimension of the formal, or the formal a dimension of the empirical. Quine's naturalization of epistemology and his doctrine of the reciprocal containment of epistemology and ontology entail that the philosophical investigation into the scientific representation of the world be carried out from within the ontological framework provided by science itself. As we shall see in section iii, logical empiricism appropriates the scientific (i.e., logico-mathematical) concept of model but overcodes it in terms of the formal/empirical distinction in such a way that it is no longer the formal that models the empirical (as in vulgar empiricism) but rather the empirical that models the formal (as it does for Carnap). Quine's naturalization of epistemology and subversion of the analytic/synthetic distinction reveals one way of overcoming this formal/empirical dichotomy without relinquishing empiricism: by grounding the formal modelling of the empirical in an empirical modelling of the formal. In other words, by proposing as ultimate horizon for naturalized epistemology the construction of a scientific model of science's model-constructing capacity in general. For Badiou in *Concept of Model*, writing in the late 1960s, this strategy is exemplified by the conjunction between cybernetics and empiricism, and, more specifically, by the integration of AI research into the programme of naturalized epistemology spawned by Quine.[31] The goal of the latter consists in explaining the congruence between the world and its scientific representation by elaborating a scientific theory of representation. The scientific modelling of reality is explained in terms of neurocomputational processes which are themselves part of science: scientific representation is integrated into the science of representation (which is itself a representation of science).[32] Thus in a surprising empiricist mimesis of the serpent of absolute knowledge swallowing its own tail, naturalized epistemology seeks to construct a virtuous circle wherein the congruence between fact and form is explained through the loop whereby representation is grounded in fact and fact is accounted for by representation. As Badiou puts it: "If science is an imitative artifice [*artisanat*], the artificial imitation of this artifice is, in effect, Absolute Knowledge."[33] Thus the ideological deployment of the category of model allows empiricism to progress from positivism to a pragmatist variety of absolute idealism.

But for Badiou this pragmatist idealism and the empiricist representation of representation concomitant with it remain beholden to ideological doxas which, despite the latter's reflexivity, they are incapable of registering. The idealization of science as imitative artifice occludes its reality as process of cognitive production, which Badiou maintains should not be understood in terms of a confrontation between formal operations and a pre-existing empirical reality but rather as a theoretical practice developing demonstrations and proofs within a determinate historical materiality whose structural specificity is itself the object of a science: historical materialism. Only in light of the latter does epistemology become sensitive to the relation between science and its ineliminable ideological representation without relapsing into historicism (which is itself a variant of empiricism for Badiou). Moreover, the pragmatist usage of the category of model elides the distinction between cognitive production and the technical regulation of concrete processes. This latter elision is exemplified by economics, where the use of models consists in passing off the discipline's own technical subservience to conditions of production as the timeless necessity of a specific type of economy, whose benefits the models exemplify. Here again, Badiou insists, the task of materialist epistemology consists in exposing the representational idealization of science as imitative artifice by providing an account of the autonomy of scientific practice vis-à-vis its ideological representation while acknowledging its constitutive yet non-empirical historicity.

Ultimately, every variant on the fundamental theme of empiricism, which consists in the difference between fact and form, is faced with the problem of how to articulate the unity of that difference. For vulgar empiricism, the unity of the duality of fact and form is posed in terms of the question of the model's reproduction or functional simulation of reality. Thus an extrinsic relation of analogical resemblance is invoked in order to bridge the gap between the supposedly inert opacity of empirical fact on the one hand, and the active construction of theoretical form on the other. Here, of course, the precise nature of the desired "resemblance," "simulation," or "reproduction" remains vague and ambiguous. For the brand of pragmatist idealism spearheaded by Quine, however, the unity of the difference can be unearthed by sealing the gap, by replacing congruence with reciprocal presupposition, by supplanting "resemblance" and "simulation" with isomorphy, and by ensuring the double articulation of fact and form. No longer inert and passive, the structure of the empirical itself generates the form of representation that will account for it. Here, evolutionary epistemology and ultimately natural history provide the explanatory fulcrum for explaining the relation between empirical fact and theoretical form.

As we shall see in section iv, Badiou's own stance in *Concept of Model* exhibits both surprising parallels and profound divergences with Quine's. Like Quine, Badiou insists on philosophy's dependence upon science and on the immanent autonomy of scientific thought. Like Quine, he refuses any recourse to a science-transcendent philosophical foundationalism. But unlike Quine, Badiou will have no truck with naturalism and hence refuses to reintegrate the sciences (i.e., mathematics) into a broader evolutionary and ultimately biological narrative about the development of human cognitive prowesses. For Badiou, the irreducible variety of scientific practices each harbour discontinuous historicities that remain internal and immanent to each of them; historicities which cannot be reabsorbed into an all-encompassing bio-evolutionary narrative about the human organism's "science-forming" faculties. In this regard, notwithstanding a justifiable aversion to spuriously "totalizing" evolutionary narratives, and even though viable evolutionary accounts of the mathematical sciences (arguably the most spectacular manifestation of human cognitive prowess) remain a very distant prospect, Badiou's apparent refusal to countenance any mediating nexus between natural history and the science of history betrays an all-too-ideological antipathy to biology – as though in spite of Darwin, biology still harboured too many residues of its Aristotelian inception for even a heterodox Platonist to stomach. But however unappetizing the prospect of a naturalized epistemology may be to Badiou in its pragmatist idealist guise, once one has discounted transcendentalism, as Badiou has, it becomes difficult to reconcile insistence on the autonomy of the sciences as discrete registers of cognitive production with an unqualified disdain for the one scientific discourse that is in a position to mediate between natural and cognitive production, or *phusys* and *praxis*. For is it not precisely the appeal to an absolute (theological) cleavage between two fundamentally different kinds of history, natural history and cultural history, or hyletic history and noetic history, that Darwin revoked?

iii the concept of model

Throughout this section, I will simply be recapitulating the main features of Badiou's own meticulous reconstruction of the mathematical concept of model. Since I am not qualified to judge the exactitude of Badiou's handling of the mathematical details, I shall confine myself to summary and paraphrase and reserve my own appraisal of Badiou's philosophical claims to section iv, in which I shall consider the account of the historical materiality of mathematical productivity which Badiou proposes in light of the scientific theory of modelling outlined below.

syntax and semantics

Badiou's account of the difference between the category and the concept of model provides the key to understanding how logical empiricism effectively recodes the distinction between the syntactical and semantic dimensions of logico-mathematical systems in terms of the distinction between formal and empirical science. In a given formal system, the set of rules specifying the difference between well-formed and illicit combinations of symbols, how expressions are to be formed and connected to or derived from one another, defines the system's syntactical aspect. The rules of deduction or syntax of a formal system allow one to derive theorems from an initial set of axioms. But not all the well-formed expressions in the system can be theorems, otherwise every expression would be legitimate and the rules of deduction would be redundant. Thus there must be at least one theorem which cannot be derived from the axioms by way of the rules of deduction. This is a formal requirement necessary in order to ensure the consistency of the system.[34] Moreover, in order to verify that a syntactical construction is not entirely arbitrary and that a formal system actually expresses a specific deductive structure, it is necessary to establish a relation of correspondence between expressions of the system and expressions belonging to a well-defined domain of "objects." Obviously, neither analogy nor resemblance suffices when it comes to defining this relation; what are required are well-defined rules of correspondence. Everything pertaining to these correspondence rules will relate to the semantics or the interpretation of the system. Given this characterization of semantics, "meaning" has a purely extensional character: to talk of the semantics or meaning of a system is to talk of its various interpretations as governed by these rules of correspondence. Once one has defined the rules of semantic correspondence for a system, one possesses the basic requirement for constructing the concept of model. This is that every deducible expression (or theorem) of the system be linked to a "true" expression in the structure that serves as its domain of interpretation. Badiou emphasizes that the use of the term "true" in this logico-mathematical context is not intended to carry any ideological-philosophical baggage; it is simply defined in terms of a functional division enforced by a formal mechanism that invariably distinguishes between two classes of expression: "true" (or "demonstrable" or any other equivalent scientific valence) statements on the one hand; "false" (or "indemonstrable" etc.) statements on the other. If every deducible expression in the system can be made to correspond to a "true" statement in the domain of interpretation, then the latter is effectively a *model* of the formal system. The reciprocal of this claim is stronger: if for every true statement in the model there corresponds a deducible expression in the system, then the system is said to be "complete" for this particular model.[35] There is, in effect, a whole gamut of semantic properties which can be studied using mathematics: in so doing one effectively catalogues the properties of the scientific concept of model.[36]

Logical empiricism exports this concept into epistemology by characterizing science's purely mathematical or "formal" dimension as its syntactical aspect, and by designating its experimental or "material" aspect as a semantic interpretation of its formal dimension. Whereas science's formal/theoretical dimension is said to be governed by the demands of consistency, its empirical/experimental aspect is said to necessitate an examination of concrete models. Experimental apparatuses are at once instruments for constructing such models and the realm within which to deploy the rules of correspondence between formal calculation and concrete measurement.[37] The choice of scientific theory is constrained by the experimental model and correspondence rules on the one hand, and by the system and its syntactical rules on the other. Thus, in Carnap's *Meaning and Necessity*,[38] for example, science is structured by the interplay between the constraint of syntactic deduction and the exactitude of semantic interpretation. But for Carnap, unlike vulgar epistemology, it is the empirical that functions as a model for syntactical artifice, rather than the reverse. Whereas the vulgar epistemological category of model is forced to rely on para-theoretical notions of "resemblance," logical empiricism is able to characterize the appropriate epistemological criteria governing the relation between science's formal and empirical aspects in terms of demonstrable theoretical properties such as that of syntax's completeness relative to a given model. Nevertheless, we shall see below why this is still not enough to legitimate its appropriation of the concept of model.

system and structure

The three prerequisites for the concept of model are: (1) a formal system comprising a (finite) set of symbols (logical operators, individual constants [a, b, c . . .], predicates [P, Q, R . . .], individual variables [x, y, z . . .]) + rules of formation + rules of deduction + a list of axioms; (2) a structure which provides the domain of interpretation for the system, and which is defined as a non-empty set V comprising a list of individual elements and a list of subsets + two supplementary marks "True" and "False"; (3) a correspondence function ζ mapping individual constants of the system to some element of V, and predicative constants of the system to some subset of V.

With regard to the above, there are three important remarks to be made. First, the use of the concept of set is absolutely decisive. The vague notion of a "domain of objects" is dangerously equivocal and tainted with empiricism. Only if the notion of interpretative domain is cashed out in terms of the mathematical concept of set can the concept of model be scientifically articulated. The scientific status of "semantics," and hence of the concept of model, depends upon the former being established within an existing branch of mathematics, so that the rules governing the interpretation of a formal mathematical system are formulated within a (non-formal) branch of mathematics itself.

Second, the stipulation that the list of formal symbols comprised by the system be finite means that they should be denumerable using natural whole numbers. This is another indispensable requirement for the construction of the concept of model. Every well-formed expression of the system should consist of a denumerable or, in the case of most systems, finite series of indecomposable symbols. It is a condition for the scientific theory of models that the formal language of the system not be continuous. Just as the recourse to set-theory proved necessary for the scientific characterization of semantics, there is an unavoidable recourse to the mathematics of whole numbers (and to recursive arithmetic in particular) in the conception of syntax. The deployment of a scientific concept of model necessarily presupposes the existence (or "validity") of these mathematical practices. This explains the remark which served as the first of our two epigraphs by Badiou: "One establishes oneself in science from the start. One does not reconstitute it from scratch. One does not found it."

Third, it is important to note the intra-syntactical distinction between the system's logical and mathematical axioms. A logical axiom is one whose functioning depends solely on the logical connectives which figure in it and remains unaffected by substitution of the fixed constants (individual or predicative) contained in it. A mathematical axiom is one which singularizes at least one of the fixed constants that figure in it by separating it from at least one other; thus it is sensitive to their substitution.

Given these prerequisites, the construction of the concept of model proceeds in the following way. Using the set-theoretical resources of the structure and the correspondence function ζ, one defines the validity and invalidity of a well-formed expression of the system relative to the structure. One then specifies the conditions under which a particular structure is a model for the system by establishing a relation between syntactic deducibility (i.e., the fact that an expression A is a theorem of the system) and semantic validity (i.e., the fact that A is valid for a structure, or a particular type of structure, or even any structure whatsoever). A "closed instance" of the expression A is an expression of the type A(a/x) (b/y) (c/z), wherein all of A's free variables have been replaced by constants. The following definition of "validity" can then be proposed: an expression A in the system is valid for a structure if, for every closed instance A′ of A, one obtains A′ = True for that structure. More particularly, a closed expression A is valid if A = True since it has no other closed instances apart from itself (nothing in it is substitutable).[39] One can then use this definition of validity to demonstrate that if the axioms of a system are valid, all the theorems of that system are also valid. In effect, since a deduction begins with an axiom, and subsequently comprises nothing but axioms or expressions derived from previous expressions via the application of the rules of deduction, then, if the axioms are valid, every expression used in the deduction is also valid. Thus the correspondence function ζ which sustains the procedures of evaluation allows us to infer from the syntactic concept of deducible statement (theorem) the semantic concept of statement-valid-for-a-structure. This allows Badiou to propose the following definition of model: "A structure is the model of a formal theory if all the axioms of that theory are valid for that structure."[40]

the logical and the mathematical

The distinction between logical and mathematical axioms can be characterized semantically in terms of the scope of their respective validities: whereas logical axioms are valid for every structure, a mathematical axiom is valid only for particular structures. Thus, from a semantic point of view, logic is equivalent to the "systematicity" of structure as such, whereas mathematics is equivalent to the theory of the types of structure.[41] But this is not to say that logic enjoys some putatively "trans-historical" status as condition of possibility for mathematical rationality as such; or that it must always already be there as the condition for, rather than the result of, the history of reason. In order to overcome this dichotomy between logicist transcendentalism and historicist relativism, Badiou suggests that logic itself be conceived as doubly articulated between syntactic system and semantic structure. The opposition between history and a priori is circumvented by the relation of reciprocal presupposition between the logical practice inherent in every semantic demonstration and the experimental construction of particular logical systems. Thus the "trans-historicity" of logic can be scientifically accounted for in terms of the experimental property whereby a purely logical system, all of whose axioms are logical, contains no semantic indication of its models. Since every structure is a model for such a system, the concept of model is not logically discernible from that of structure for it. Accordingly, it is mathematical axioms that suspend this semantic indiscernibility of logical structure by effectuating the inscription of a structural gap between syntactic system and semantic structure; a gap within which the concept of model comes into play. Thus the concept of logic neither transcends nor subsumes mathematics; it remains inseparable from the couple which it forms with the latter. The contrast between the logical and the mathematical is a syntactical redoubling of the semantic distinction between structure and model. Given two structures whose difference is indexed by the fact that one of them is a model for a given formal system while the other is not, it becomes possible to classify the axioms of the system into those that are logical and those that are mathematical. The former index the unity of system and structure while the latter index their difference.[42] Thus "[a] model is the mathematically constructible concept of the differentiating power of a logico-mathematical system."[43]

experimentation and demonstration

The double occurrence of the term "mathematics" in the formulation quoted above indexes the way in which the means of mathematical production, namely the conjunction of experimentation and demonstration, are themselves mathematically produced. This double articulation cannot be transplanted outside of mathematics or duplicated in the relation between a supposedly formal theory and its putatively material instantiation. Thus, given the manner in which the construction of the concept of model depends at every step on the double articulation between two particular branches of mathematics, namely recursive arithmetic and set-theory, it is profoundly misleading to claim, as some philosophers do, that the concept of model indexes formal thought's relation to its (empirical/material) "exterior." Extra-systemic (structural) inscriptions are only capable of providing a domain of interpretation for those of the system according to a mathematical envelopment which subordinates the former to the latter. Syntax is an arithmetical discipline; semantics a set-theoretical one.[44] The theory of the apparatuses of inscription conceived as mathematical objects is arithmetical. It allows one to engage in an ordering and inductive numbering of the experimental set-up; to evaluate its power and complexity through reasonings bearing on the structure of inscription which the system either allows or prohibits. By way of contrast, the theory of the usage of these apparatuses, conceived as experimental operations, classifies the regions of the mathematical material which is to be processed by the apparatus; this is the aim of the concept of structure, which is itself produced by the most general, most all-enveloping mathematical theory produced thus far: set-theory.[45]

The sole basis for the unity-in-difference of syntax and semantics lies in the intra-mathematical relation between the arithmetical material and the set-theoretical material. Any attempt to export the concept of model outside the mathematical realm violates this necessarily intra-mathematical relation and is thus illegitimate. In this regard, logical empiricism's attempt to export the mathematical concept of model into epistemology is doubly illegitimate. First, because it tries to theorize science in general on the basis of a difference between syntax and semantics that is merely an ideological distortion of the regional and intra-mathematical distinction between recursive arithmetic and set-theory. Second, because its conception of empirical facts as models for formal theories is merely an *analogon* for the intra-mathematical correspondence between system and structure and fails to register the crucial respect in which the modelling of a formal system constitutes a means for experimenting upon and ultimately transforming the rigour or generality of that system. This perpetual labour of intra-mathematical transformation becomes inoperative in cases where the putative domain of interpretation is not already mathematical and hence semantically assignable as capable of corresponding to a syntactical apparatus.

Thus, contrary to what the ideological appropriation of the theory of modelling suggests, scientific practice does not proceed from formal theory to concrete model, or from system to structure, but from structure to system, from model to theory and back again in ceaseless dialectical interplay. The historical reality of mathematical production does not confront us with the challenge of testing the theory through the model, but of testing the model by means of the theory. The problem faced by actually existing mathematical practice, as opposed to its epistemological idealization, consists in specifying the formal theory modelled by the historical plurality of structures, in identifying the appropriate syntactic signature – i.e., the formal theory – for a given type of structure. This is the problem of mathematical formalization. If "[s]emantics is an experimental protocol,"[46] this is not, Badiou insists, because a model is an experimental realization of a formal system, but because, on the contrary, it is a structure embodying a conceptual demonstration whose experimental verification is carried out by means of inscription in a formal syntax. The formal system is retrospectively constituted as the experimental moment, the material linkage of verification, in the wake of the conceptual demonstration articulated in the model. Thus formalized syntaxes are materialized theories: means of mathematical production just like the vacuum tube or particle accelerator for physics: "It is precisely because it is itself a materialized theory, a mathematical result, that the formal apparatus is capable of entering into the process of the production of mathematical knowledge; a process in which the concept of model does not indicate an exterior to be formalized but a mathematical material to be tested."[47] It is the system that is formalized by means of the demonstration provided by the model. Formalization proceeds through the experimental verification of the conceptual demonstration provided by the model. The latter provides the material to be tested by its inscription in a formal system:

> [T]he philosophical category of effective procedure, of what is explicitly calculable through a series of unambiguous scriptural manipulations, lies at the heart of all mathematical epistemology. This is because this category distils the properly experimental aspect of mathematics, that is to say, the materiality of its inscriptions, the montage of notations [...] Mathematical demonstration is tested

> [*s'éprouve*] through the rule-governed transparency of inscriptions. In mathematics, inscription represents the moment of verification.[48]

Thus the formal system provides the instrument of experimental inscription required in order to verify the conceptual demonstration deployed in the structure of the model; but this verification in turn becomes a means of formalization. Demonstration and formalization are bound together in a material dialectic of reciprocal presupposition. It is this dialectic, and the mechanism of inscription upon which it depends, that lie at the heart of Badiou's claims about the historical materiality of mathematical production.

iv the historicity of mathematical production

Badiou's reconstruction of the concept of model provides the basis for the construction of a category of model which is to be mobilized within a dialectical-materialist account of the historicity of scientific practice. Badiou is explicit about the structure of his argument: there can be no question of using the concept of model as the basis for a theory of mathematical historicity, which would amount to a transparently ideological misappropriation of a scientific concept; rather, it is a question of deploying an explicitly materialist category of model on the basis of a theory of the historicity of mathematical science already implicit in Badiou's preliminary critiques of the notional uses and abuses of the concept of model.

The "materiality" of mathematical practice is not to be understood as an analogue of the inexistent philosophical category "matter," but rather as an index of the scriptural production of difference. This account of scriptural materiality is, so to speak, the esoteric subtext of Badiou's materialist epistemology of science. It has to be reconstructed on the basis of various suggestive but elliptical hints scattered throughout *Concept of Model* and another roughly contemporaneous (i.e., 1967–69) text, the extraordinary "Mark and Lack: About Zero."[49] At the conclusion of the latter, Badiou writes: "Science is the veritable archi-theatre of writing: traces, crossed out traces, traces of traces; movement wherein there is not the slightest prospect of recentering the detestable figure of Man: the sign of the nothing."[50] Badiou's recurrent emphasis on the materiality of logico-mathematical inscription in these early epistemological writings seems to effect a critical conjunction of Lacan's theses about the agency of the letter and Derrida's claims about the disseminatory force of archi-writing. Logico-mathematical inscription circumvents the metaphysical primacy of the linguistic signifier via a "stratified multiplicity"[51] of differential traces which "no signifying order can envelop";[52] one which pulverizes the presence of the object – "Neither the thing nor the object has any more existence here than has their traceless exclusion"[53] – and dissolves the unity of the subject:

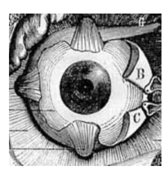
> [T]here is no subject of science. Infinitely stratified, adjusting its transitions, science is a pure space, without a reverse or mark or place of what it excludes. It is foreclosure, but foreclosure of nothing, and so can be called the psychosis of no subject,[54] hence of all; fully universal, shared delirium, one only has to install oneself within it to become no-one, anonymously dispersed in the hierarchy of orders. Science is an Outside without a blind-spot.[55]

Here, we encounter all the essential features of Badiou's Platonist materialism: scientific thought is outside, beyond the enclosure of ideological representation; not because the subject of science enjoys intuitive access to a realm of transcendent objects, but on the contrary, because the remorselessly mechanical "rule governed transparency" of logico-mathematical inscription dissolves the consistency of the object and the coherence of the subject in the infinitely stratified multiplicity of scientific discourse. The immanent autonomy of scientific discourse, its non-representational character, is a consequence of its machinic nature, since "[a] formal system *is* a mathematical machine, a machine for mathematical production, positioned within that production."[56] But the means of mathematical production are themselves produced; the mathematical machine or instrument is also a mathematical product, a result: there would be no formal systems without recursive arithmetic, and no rigorous experimental protocols for such systems without set-theory.

Yet it is because this scientific reproduction of the means of production harbours a constitutive historicity that science's self-reproduction is inherently differential. Or rather, it is the inherently differential (dialectical) nature of scientific reproduction that generates its historicity. The perpetual dialectic of demonstration and experimentation is the motor of scientific history. Scientific re-production is self-differentiating because of the way in which science itself intervenes within a determinate epistemological conjuncture by means of formal experimentation. Thus, for example, by proving the consistency of a model of axiomatic set-theory with the Axiom of Choice and the Continuum Hypothesis, Gödel demonstrates that these two axioms can be integrated into the formal theory without compromising its coherence. He thereby provides a conceptual sanction for mathematical practice: "In doing so, [Gödel's experimentation] transforms, not the theory, but the status of the theory within the historical process of the production of knowledges."[57] Given a mathematical configuration inscribed within the history of that science, to treat it as a model of a formal system is to situate its specificity by transposing it beyond the narrow ambit of the spontaneous illusions engendered by its singular production and into the wider mathematical space constituted by the various models of the system. Consequently, the experimental apparatus is a nexus of practices. The double articulation of formal experimentation and conceptual demonstration becomes the driving force for science's own epistemic interventions within determinate historical configurations. In the history of a science, the experimental transformation of practice via a determinate formal apparatus retrospectively assigns the status of model to those antecedent instances of practice. Conversely, conceptual historicity, which is to say the "productive" value of formalism, derives both from its theoretical dependency as an instrument and from the fact that it possesses models, i.e., that it is integrated into the conditions of the production and reproduction of knowledge: as Badiou states: "[s]uch is the practical guarantee of formal set-ups."[58]

For Badiou, then, the materialist category of model designates formalism's own retrospective causality upon its own scientific history, which conjoins an object (a model) and a usage (a system). The historicity of formalism consists in the "anticipatory intelligibility" of what it retrospectively constitutes as its own model.[59] Ultimately, the fundamental epistemological problem is not that of the nature of the representative relation between the model and the concrete, or between the formal and the model; rather, "[t]he problem is that of the history of formalization."[60] The materialist category of model proposed by Badiou designates the meshwork of retroactions and anticipations from which the history of formalization is woven; the history of its anticipatory cuts and its retrospective reconfigurations.[61] The historicity of scientific (re-)production is constituted by this differential meshwork of epistemic cuts and reconfigurations. Thus there is no need to invoke empirically arbitrary, para-theoretical "paradigm shifts" to account for the structural discontinuities that punctuate scientific history. Discontinuity is already inherent in the immanent conceptual mechanisms of scientific practice, for "science is precisely that which is ceaselessly cutting itself loose from its own indication in re-presentational space [i.e., ideology]."[62] This is the key to understanding the second of our introductory epigraphs: "[T]here are no crises within science, nor can there be, for science is the pure affirmation of difference."

Contra naturalism, the "science" within which Badiou recommends thought establish itself in spite of the otiose prevarications of transcendentalism cannot be mistaken for its empiricist representation or conflated with an ambient scientific worldview, a diffuse ideological distillate synthesized from various scientific disciplines (such is the composition of Neurath's boat); rather, it is an entirely autonomous, ceaselessly self-differentiating mode of theoretical practice invariably defined by a specific historical conjunction between conceptual demonstration and formal experimentation.



notes

1 *Le Concept de modèle. Introduction à une épistémologie matérialiste des mathématiques* [The Concept of Model: Introduction to a Materialist Epistemology of Mathematics] (Paris: Maspero, 1969) 42. It is important to bear in mind that, in the context in which Badiou is writing here (i.e., the context defined by the work of Althusser, Bachelard, and Canguilhem), "epistemology" refers to the "theory of science," and not the "theory of knowledge" as commonly understood in Anglo-American academic philosophy. According to the latter, epistemology is concerned principally with problems concerning the nature of rationality, belief, truth, scepticism, etc. – problems whose philosophical scope far exceeds that of "philosophy of science" proper. Thus the question about "how science represents the world" is subsumed by the larger issue concerning the precise nature of the epistemic relation between mind and world. But for Badiou, as well as for Althusser, Bachelard, and Canguilhem, such questions cannot be the concern of epistemology proper since they remain fatally enmeshed in the empiricist prejudices of representationalism, which can only obstruct proper philosophical understanding of scientific theory and practice. One suspects that further investigation into the deeper conceptual ramifications harboured by this seemingly trivial nominal difference would go a long way towards explaining the fundamental philosophical divergence between the concerns of post-Bachelardian "epistemology" in France and those of Anglo-American "philosophy of science."

2 "Marque et manque: A propos de zéro" [Mark and Lack: About Zero], *Cahier pour l'analyse* no. 10 (1969) 165. Badiou's article is actually dated "January 1967," though it was not published until 1969.

3 "Le (Re)Commencement du matérialisme dialectique," *Critique* no. 240 (1966): 438–67. Ostensibly a review article focusing on three works (Althusser's *Pour Marx* (Paris: Maspero, 1965) and "Matérialisme dialectique et matérialisme historique," *Cahiers Marxistes-Léninistes* no. 11 (Apr. 1966); and *Lire Le Capital* by Althusser, Balibar, Establet, Macherey, and Ranciére, 2 vols. (Paris: Maspero, 1965)), this early piece not only provides a magisterial critical overview of the Althusserian project but is also a powerfully original philosophical intervention in its own right.

4 *L'Être et l'événement* (Paris: Seuil, 1988). An English translation by Oliver Feltham is due to be published by Continuum in 2006.

5 *Logiques des mondes* (Paris: Seuil, 2005). For an introduction to the latter, see the "Logics of Appearance" section in Badiou's *Theoretical Writings*, eds. R. Brassier and A. Toscano (London: Continuum, 2004) 163–231.

6 Badiou, "Le (Re)Commencement du materialisme dialectique" 464.

7 "Politics and Philosophy: An Interview with Alain Badiou," *Angelaki* 3.3 (1998) 127.

8 Cf. *Being and Event*, Introduction (Paris: Seuil, 1988) 7–27.

9 Alain Badiou, "Mathematics and Philosophy: The Grand Style and the Little Style" in *Theoretical Writings*, eds. R. Brassier and A. Toscano (London: Continuum, 2004) 16.

10 Daniel Smith provides a sophisticated variant of this particular criticism whilst delivering a sharp rejoinder to Badiou's reading of Deleuze in "Badiou and Deleuze on the Ontology of Mathematics" in *Think Again: Alain Badiou and the Future of Philosophy*, ed. P. Hallward (London: Continuum, 2004) 77–93. Smith elaborates on the Deleuzian distinction between "royal" and "minor" (or "nomad") science by developing a highly illuminating contrast between the "axiomatizing" and "problematizing" tendencies in mathematical practice. He then uses this distinction as an interpretative prism through which to correct what he takes to be Badiou's misreading of Deleuze, and criticizes the former for his narrowly "royalist" conception of science and his exclusively "axiomatic" characterization of mathematics. Yet although this distinction between axiomatics and problematics – partially rooted in the work of Albert Lautman – is undoubtedly of considerable philosophical significance, it is doubtful that it can be used to defend Deleuze against Badiou in the way that Smith attempts to do here. One cannot help suspecting that the distinction between "problematics" and "axiomatics," like that between "minor" and "royal" science upon which it is based, is merely a *reiteration* (rather than an independent conceptual legitimation) of the Deleuzian distinction between intensive and extensive multiplicities (or open and closed, smooth and striated, virtual and actual, etc.); the fundamental distinction around which Deleuze's entire philosophy is

coordinated but whose necessity Badiou – regardless of the undeniable infelicities in his reading of Deleuze – is surely entitled to call into question. Invoking the rights of problematics against axiomatics in order to defend the necessity of the distinction between intensive and extensive multiplicities will prove to be of no avail if the former distinction turns out to be a restatement of the latter.

11 Indeed, there seem to be legitimate grounds for claiming that "philosophy of science" as we know it was spawned by a particular philosophical doctrine: logical empiricism. Thus in his contribution to the MIT anthology *The Philosophy of Science*, co-editor Richard Boyd declares:

> Almost all work, foundational or applied, in English-language philosophy of science during the present [twentieth] century has either been produced within the tradition of logical empiricism or has been written in response to it. Indeed it is arguable that philosophy of science as an academic discipline is essentially a creation of logical empiricists and (derivatively) of the philosophical controversies it sparked. (Richard Boyd, "Confirmation, Semantics, and the Interpretation of Scientific Theories" in *The Philosophy of Science* (Cambridge, MA: MIT P, 1991) 3)

12 Badiou, *Le Concept de modèle* 12. *The Concept of Model* originated as Badiou's contribution to a seminar series set up by Althusser entitled "Cours de philosophie pour scientifiques" [Course in Philosophy for Scientists] at the École Normale Supérieure during the 1967–68 academic year. Althusser's first four lectures in the series, to which Badiou is alluding here, were entitled "Philosophy and the Spontaneous Philosophy of the Scientists" and can be found in his *Philosophy and the Spontaneous Philosophy of the Scientists and Other Essays* (London: Verso, 1990) 69–144. Althusser's fifth lecture in the same series was entitled "Du cotée de la philosophie" [On the Side of Philosophy] and published posthumously in vol. II of his *Écrits philosophiques et politiques* [Philosophical and Political Writings] (Paris: Stock/IMEC, 1997) 265–308. Badiou may also be alluding to a passage here that occurs on page 277.

13 Here we have an example of how a thesis which has been central to Badiou's work from the beginning is not stated explicitly until much later. Thus it is not until 1998's "Platonism and Mathematical Ontology" (in *Theoretical Writings* 49–58) that Badiou explains how his conception of Platonism subverts the basic distinction between thought and object, which in Husserlian phenomenology is given a more subtle characterization in terms of the correlation between "noesis" and "noema."

14 The characterization of historical materialism as a "science" of history is obviously contentious, particularly in light of Badiou's implicit identification of mathematics with scientificity; moreover, it is one to which I believe he no longer subscribes, but I shall not question it here. Be that as it may, Badiou's emphasis on the specific mode of conceptual "productivity" which he discerns in mathematical practice is obviously tied to the central role of productivity in historical materialism.

15 Badiou, *Le Concept de modèle* 62.

16 Badiou gives three examples: the Platonic category of "ideal number" denotes an inexistent "adjustment" between arithmetical concepts and hierarchical moral-political notions; the Kantian categories of "space" and "time" combine Newtonian concepts with notions that are relative to human faculties; and the Sartrean category of "History" combines Marxist concepts with metaphysico-moral notions such as temporality, freedom, etc. Regarding the second of these examples, it should go without saying that Badiou is using the term "category" here in a sense entirely distinct from Kant's and is perfectly well aware that for Kant space and time are "forms of intuition" rather than "categories."

17 As we shall see, the true philosophical index of "materiality" in this conjunction between historical and dialectical materialism is that of the "productivity" of a given theoretical practice. As Badiou states: "The *reality* of the epistemological materialism which I am trying to introduce here is indissociable from an effective practice of science" (*Le Concept de modèle* 29).

18 Badiou, *Le Concept de modèle* 12.

19 The use of the term "presence" here is perhaps intended as an allusion to Derrida's work, with which Badiou was certainly already familiar (cf., for instance, Badiou, "Le (Re)Commencement du matérialisme dialectique" 445); one possible implication being that the deconstruction of logocentrism can be enlisted as part of the critique of empiricist epistemology.

20 Badiou, *Le Concept de modèle* 9.

21 Reprinted in *The Philosophy of Science*, eds. R. Boyd, P. Gasper and J.D. Trout (Cambridge, MA: MIT P, 1991) 393–404.

22 Carnap is careful to distinguish between unification couched in terms of the logical reducibility of terms, which he endorses, and unification understood as the derivation of the laws of one science (e.g., biology) from those of another (e.g., physics), about which he expresses reservations:

> [T]here is a common language to which both the biological and the physical laws belong so that they can be logically compared and connected. We can ask whether or not a certain biological law is compatible with the system of physical laws, and whether or not it is derivable from them. But the answer to these questions cannot be inferred from the reducibility of the terms. At the present state of the development of science, it is certainly not possible to derive the biological laws from the physical ones. Some philosophers believe that such a derivation is forever impossible because of the very nature of the two fields. But the proofs attempted so far for this thesis are certainly insufficient. (Carnap, "Logical Foundations of the Unity of Science" in *The Philosophy of Science*, eds. R. Boyd, P. Gasper, and J.D. Trout (Cambridge, MA: MIT P, 1991) 403)

23 In *From a Logical Point of View*, 2nd ed. (Cambridge, MA: Harvard UP, 1980) 20–46.

24 Willard Van Orman Quine, "Two Dogmas of Empiricism" in ibid. 22.

25 Ibid. 43.

26 Donald Davidson, "On the Very Idea of a Conceptual Scheme" in *Enquiries into Truth and Interpretation* (Oxford: Clarendon, 1984) 183–98. Although I cannot do so here, it would be instructive to compare and contrast Davidson's critique of Quine's scheme/content dualism with Badiou's critique of empiricism. Despite the semblance of a shared antipathy to empiricism, Badiou's definition of the latter is wider ranging than Davidson's and I suspect the latter's work would still seem all too empiricist to Badiou.

27 Quine, "Two Dogmas of Empiricism" 45–46.

28 Ibid. 44.

29 Ibid. 44.

30 Badiou, *Le Concept de modèle* 11.

31 A conjunction which birthed cognitive science, and whose most distinguished contemporary representative is arguably Daniel Dennett.

32 See, for example, Paul Churchland, *A Neurocomputational Perspective: The Nature of Mind and the Structure of Science* (Cambridge, MA: MIT P, 1989), or more recently, Jean-Pierre Changeux, *The Physiology of Truth: Neuroscience and Human Knowledge* (Cambridge, MA: Harvard UP, 2004).

33 Badiou, *Le Concept de modèle* 21.

34 Badiou examines this fundamental formal requirement at length in the appendix to *Concept of Model* 69–90.

35 In his famous theorem of 1931, Gödel demonstrated the incompleteness of the formal system of arithmetic, i.e., of a formal system capable of being modelled by "classical" or recursive arithmetic, by showing how its model contains a true statement for which there is no corresponding deducible theorem in the system. Thus a system may be consistent but incomplete, or complete but inconsistent, but it cannot be both consistent and complete. Cf. "On Formally Undecidable Propositions of *Principia Mathematica* and Related Systems" reprinted in *Frege and Gödel: Two Fundamental Texts in Mathematical Logic*, ed. Jean Van Heijenoort (Cambridge, MA: Harvard UP, 1970) 87–107.

36 Cf. Badiou, *Le Concept de modèle* 24–25.

37 Cf. ibid. 25.

38 Rudolf Carnap, *Meaning and Necessity* (Chicago: U of Chicago P, 1956).

39 Cf. Badiou, *Le Concept de modèle* 41.

> It should be noted that this procedure is constructed by means of recurrence over the "length" of expressions, i.e. over the *number of symbols* which constitute them. One begins with elementary expressions of the type P(a), which are directly evaluated in the structure, by examining the eventual belonging of a's semantic "representative" to the subset of the universe represented by P. One then adjusts the procedure which

> allows one to evaluate an expression A on the basis of the (supposedly acquired) evaluation of the shorter expressions contained in A, or contained in its closed instances. Thus the evaluation of ~B is carried out on the basis of the evaluation of B, while that of (Ex)B is carried out on the basis of B(a/x), etc. The conviction that these rules guarantee the existence of an evaluation for an expression of any length whatsoever amounts to admitting the legitimacy of reasoning by recurrence over whole numbers (in this case, over the number of symbols that enter into the composition of an expression). (Badiou, *Le Concept de modèle* 42)

40 Ibid. 44.

41 Ibid. 44–45.

42 Ibid. 47–48.

43 Ibid. 52.

44 Ibid. 55.

45 Ibid. 55–56. Significantly, Badiou also mentions category-theory here as a potential rival to set-theory in terms of all-enveloping generality.

46 Badiou, *Le Concept de modèle* 48.

47 Ibid. 58.

48 Ibid. 34.

49 Cf. n. 2 above. Not least among this text's many extraordinary features is its remarkably illuminating analysis of Gödel's famous incompleteness theorem, and its penetrating critique of certain popular philosophical misinterpretations of Gödel's work.

50 Badiou, "Marque et manque" 164.

51 Cf. ibid.

52 Ibid. 163.

53 Ibid. 156.

54 An allusion to Jacques-Alain Miller's claim that "[E]very science is structured like a psychosis" in "L'Action de la structure" [The Action of Structure], *Cahiers pour l'analyse* no. 9 (1968); reprinted in Miller's *Un début dans la vie* (Paris: Le Promeneur, 2001) 57–79.

55 Badiou, "Marque et manque" 162.

56 Badiou, *Le Concept de modèle* 54.

57 Ibid. 64.

58 Ibid. 67.

59 Ibid. 67.

60 Ibid. 68.

61 Ibid. 68.

62 Badiou, "Marque et manque" 165.

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